W-1/20

EECS 16B Section 1B

Main Topic: Intro + 16A Review

Administrivia:

- Who am I?
- Overview of EECS 16B
- Questions?

Agenda:

- Circuit Relationships
    - $V = IR$
    - KCL / KVL
    - Q1: KVL / KCL Review
- Op-Amps
    - Golden Rules
    - Q2: Op-Amp Summer
- Capacitors
    - $Q = CV$
    - $I_c(t) = C \frac{d}{dt} V_c(t)$
    - Q3: Current Sources and Capacitors
- OPTIONAL - Q4: Linear Algebra Review
    - Eigenvalues, span, nullspace

Recapping Resistors

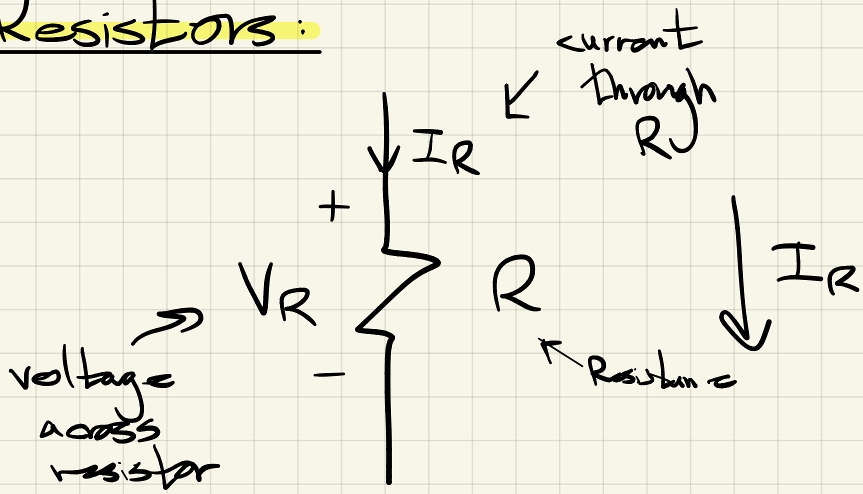
Units:

Voltage (V, v) Unit = Volts V

Current (I, i) Unit = Amperes A

Resistance (R) Unit = Ohms Ω

Resistors:



$$V = IR$$

$$R = \frac{V}{I}$$

$$I = \frac{V}{R}$$

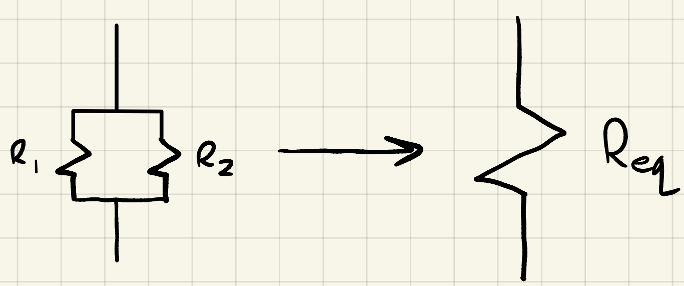
Combining Resistors:

In Series:

$$R_{eq} = R_1 + R_2$$

$$= \Sigma R_i$$

In Parallel:



$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$= \frac{\text{"Product"}}{\text{"Sum"}} = \frac{\text{"}\times\text{"}}{\text{"}+\text{"}}$$

1 KVL/KCL Review

Kirchhoff's Circuit Laws are two important laws used for analyzing circuits. Kirchhoff's Current Law (KCL) says that the sum of all currents entering a node must equal 0. For example, in Figure 1, the sum of all currents entering node 1 is $I_1 - I_2 - I_3 = 0$. Assuming that I_1 and I_3 are known, we can easily obtain a solvable equation for V_x by applying Ohm's law: $I_1 - \frac{V_x}{R_1} - I_3 = 0$.

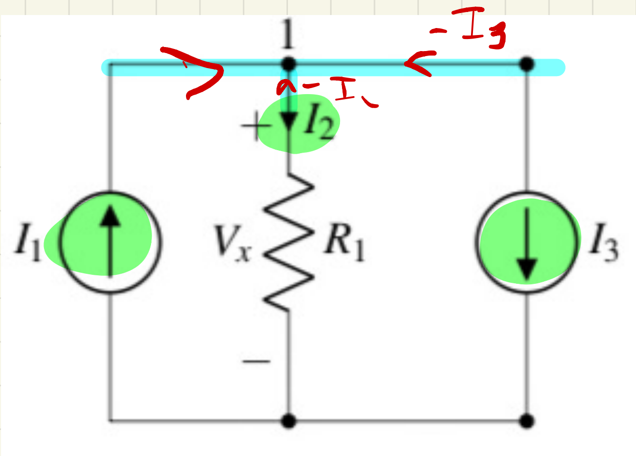


Figure 1: KCL Circuit

$\Sigma I_{in} = 0$ $\Sigma I_{out} = 0$

$I_1 = I_2 + I_3$

$I_1 - I_2 - I_3 = 0$

"Glue together components"

$V_x = I_2 R_1$

$I_2 = \frac{V_x}{R_1}$

Kirchhoff's Voltage Law (KVL) states that the sum of all voltages in a circuit loop must equal 0. To apply KVL to the circuit shown in Figure 2, we can add up voltages in the loop in the counterclockwise direction, which yields $-V_1 + V_x + V_y = 0$. Using the relationships $V_x = i \cdot R_1$ and $i = I_1$, we can solve for all unknowns in this circuit. You can use these two laws to solve any circuit that is planar and linear.

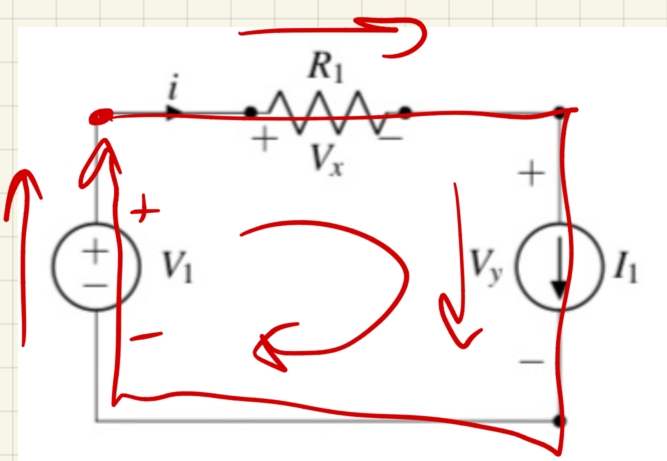


Figure 2: KVL Circuit

$\Sigma V_{loop} = 0$

$+ \rightarrow -$: drop, negative

$- \rightarrow +$: rise, positive

$-V_1 + V_y + V_x = 0$

$-V_x - V_y + V_1 = 0$

1. KVL/KCL Review

Use Kirchhoff's Laws on the circuit below to find V_x in terms of V_{in}, R_1, R_2, R_3.

KCL: $\Sigma I_{in} = 0$

$\Sigma I_{out} = 0$

KVL: $\Sigma V_{loop} = 0$

$V = IR$

$V^+ - V^- = IR$

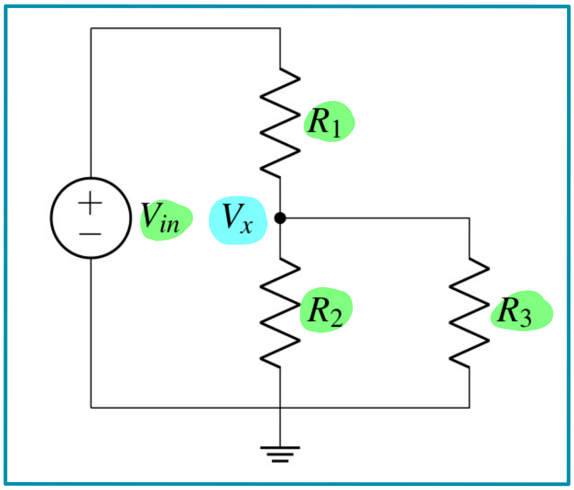
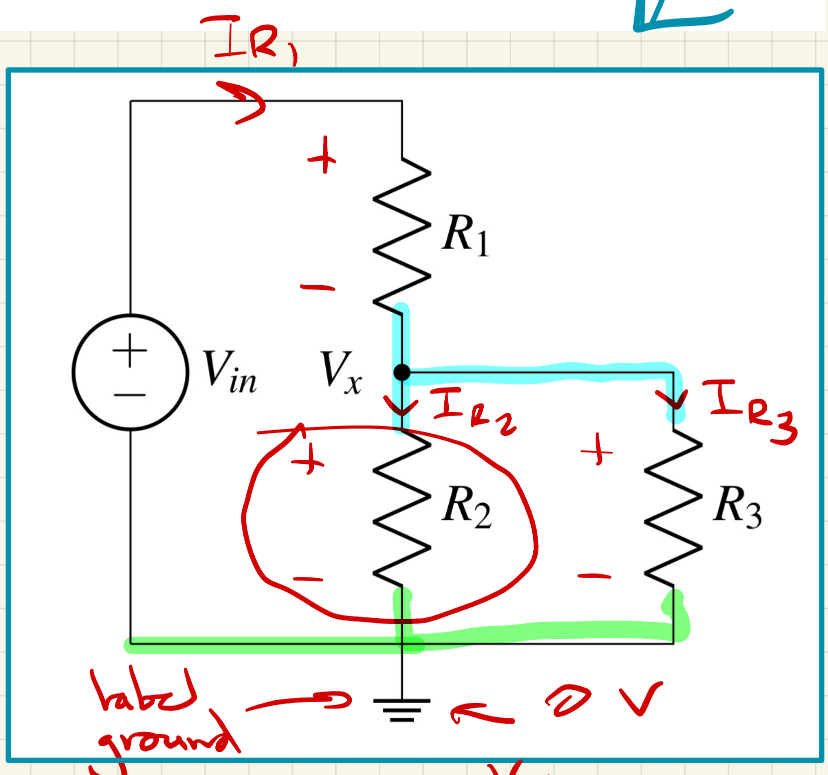


Figure 5: Example Circuit

(a) What is V_x?

(b) As $R_3 \to \infty$, what is V_x? What is the name we used for this type of circuit?



(a)

$I_{R_1} = I_{R_2} + I_{R_3}$

R_1: $V_{R_1} = I_{R_1} R_1$

$V_{in} - V_x = I_{R_1} R_1$

$\frac{V_{in} - V_x}{R_1} = I_{R_1}$

R_2: $V_{R_2} = I_{R_2} R_2$

$V_x = I_{R_2} R_2$ → $I_{R_2} = \frac{V_x}{R_2}$

R_3: $V_x = I_{R_3} R_3$ → $I_{R_3} = \frac{V_x}{R_3}$

$$\frac{V_{in} - V_x}{R_1} = \frac{V_x}{R_2} + \frac{V_x}{R_3}$$

$$R_2 R_3 (V_{in} - V_x) = R_1 R_3 V_x + R_1 R_2 V_x$$

$$V_x = V_{in} \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$\left(\underbrace{\frac{V_{in}-V_x}{\cancel{R_1}}}_{R_2R_3} = \underbrace{\frac{V_x}{\cancel{R_2}}}_{R_1R_3} + \underbrace{\frac{V_x}{\cancel{R_3}}}_{R_1R_2}\right) R_1R_2R_3$$

$$R_2R_3(V_{in}-V_x) = R_1R_3V_x + R_1R_2V_x$$

$$R_2R_3V_{in} \underline{-R_2R_3V_x} = \text{"}$$

$$R_2R_3V_{in} = R_1R_3\underline{V_x} + R_1R_2\underline{V_x} + R_2R_3\underline{V_x}$$

$$R_2R_3V_{in} = V_x\underline{(R_1R_2 + R_1R_3 + R_2R_3)}$$

$$\boxed{V_x = V_{in}\frac{R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}}$$

(b) "What if $R_3 \to \infty$?"

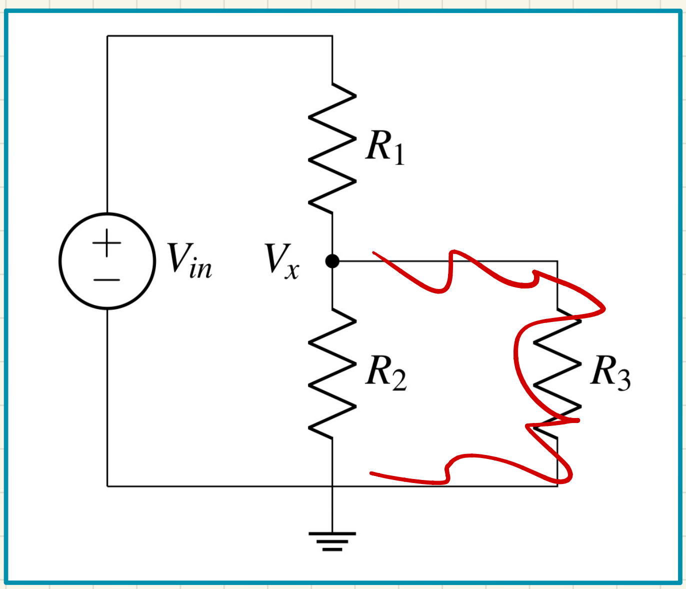
Method 1: Calculus

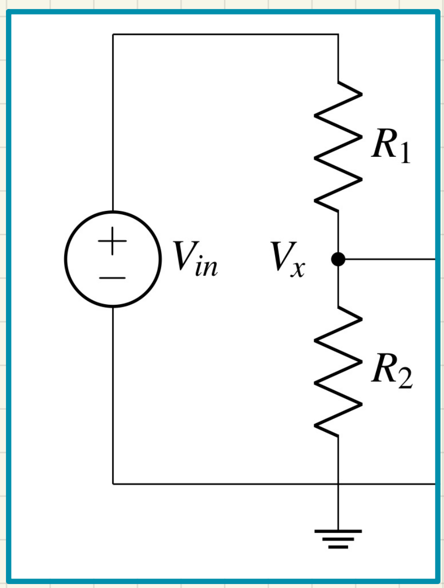
$$\lim_{R_3 \to \infty} V_x = \lim_{R_3 \to \infty} V_{in} \frac{R_2 R_3}{\cancel{R_1 R_2} + R_1 R_3 + R_2 R_3}$$

$$= V_{in} \frac{R_2 \cancel{R_3}}{R_1 \cancel{R_3} + R_2 \cancel{R_3}}$$

$$V_x = V_{in} \frac{R_2}{R_1 + R_2}$$

Method 2: Physics





$R_3 \to \infty$, "Air", "open circuit"

2 Op-amp Review

Figure 3 shows the equivalent model of an op-amp. It is important to note that this is the general model of an op-amp, so our op-amp golden rules cannot be applied to this unless certain conditions are met.

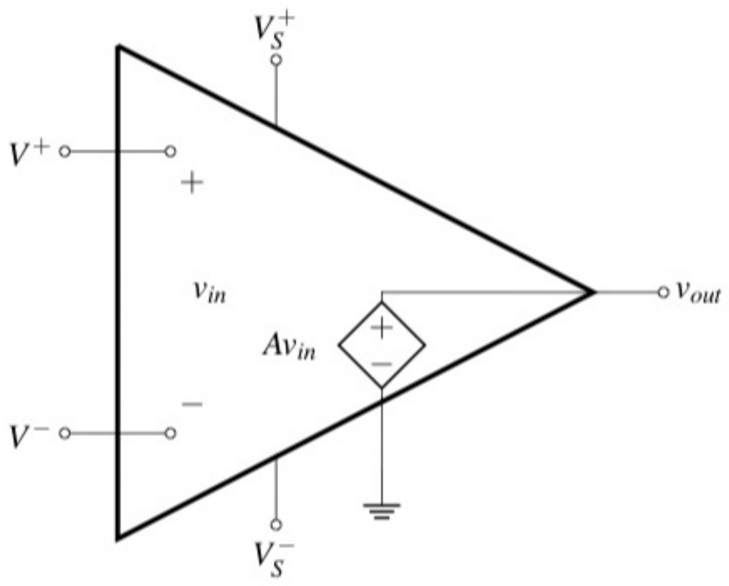


Figure 3: General Op-Amp Model

Conditions Required for the Golden Rules:

(a) $R_{in} \to \infty$

(b) $R_{out} \to 0$

(c) $A \to \infty$

(d) The op-amp must be operated in negative feedback.

When conditions (a)-(c) are met, the op-amp is considered ideal. Figure 4 shows an ideal op-amp in negative feedback, which can be analyzed using the Golden Rules.

$V_o = V^+ - V^-$

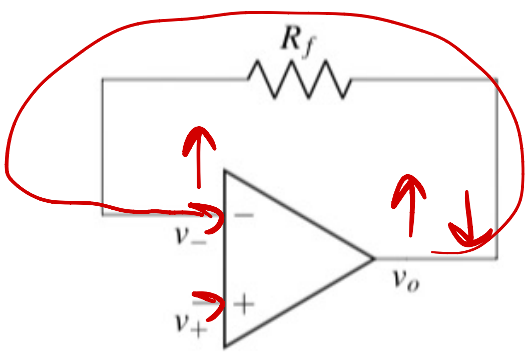


Figure 4: Ideal Op-Amp in Negative Feedback

Golden Rules of ideal op-amps in negative feedback:

(a) No current can flow into the input terminals ($I_- = 0$ and $I_+ = 0$).

(b) The $(+)$ and $(-)$ terminals are at the same voltage ($V_+ = V_-$). $V^+ = V^-$

If you would like to review these concepts more in-depth, you can check out op-amp introduction and op-amp negative feedback from the EECS16A course notes.

2. Op-Amp Summer

Consider the following circuit (assume the op-amp is ideal):

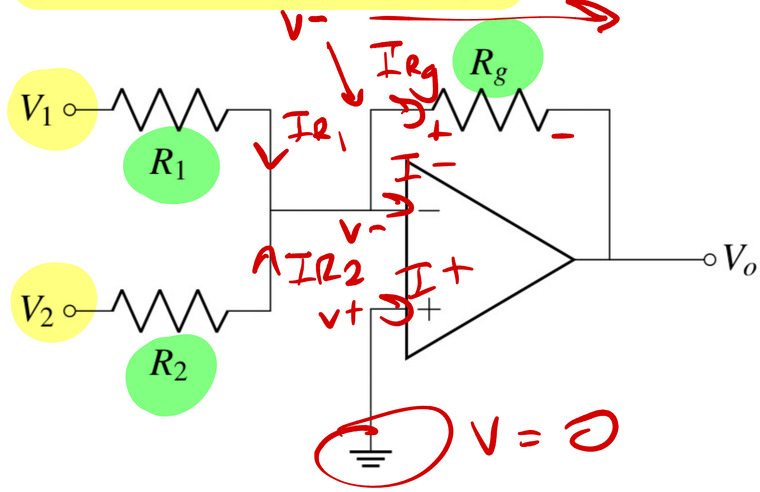


Figure 6: Op-amp Summer

What is the output V_o in terms of V_1 and V_2? You may assume that R_1, R_2, and R_g are known.

NFB? Yes.

$\to V^- = V^+$

$\to I^- = I^+ = 0$

$$I_{R_1} + I_{R_2} = I_{R_g} + \cancelto{0}{I^-}$$

$V = IR$

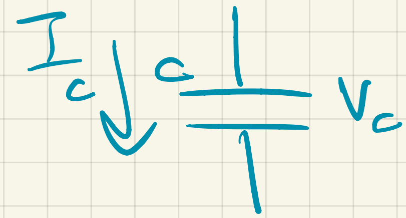
$I = \frac{V}{R}$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{0 - V_o}{R_g}$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_o}{R_g}$$

$$\boxed{V_o = -\frac{R_g}{R_1} V_1 - \frac{R_g}{R_2} V_2}$$

Capacitors



$$Q = C \cdot V$$

Q: Charge (Coulombs, C)

C: Capacitance (Farads, F)

V: Voltage

$$\frac{d}{dt} Q = I \qquad \frac{d}{dt}(Q = CV)$$

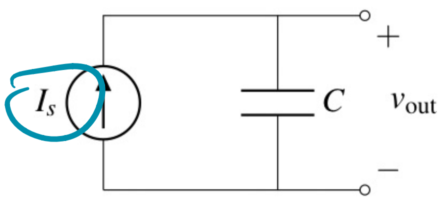
$$I_c = C \frac{d}{dt} V_c$$

3. Current Sources And Capacitors (The following problem has been adapted from EECS16A Fall 20 Disc 9A.)

Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) $= \frac{\text{Coulomb}}{\text{Volt}}$.
It may also help to note metric prefix examples: $3\,\mu\text{F} = 3 \times 10^{-6}\text{F}$.

Given the circuit below, find an expression for $v_{\text{out}}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.

$I(t)$



$$I_s = C \frac{d}{dt} V_{out}$$

$$\frac{I_s}{C} = \frac{d}{dt} V_{out}$$

Then plot the function $v_{\text{out}}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\,\mu\text{F}$.

(a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.

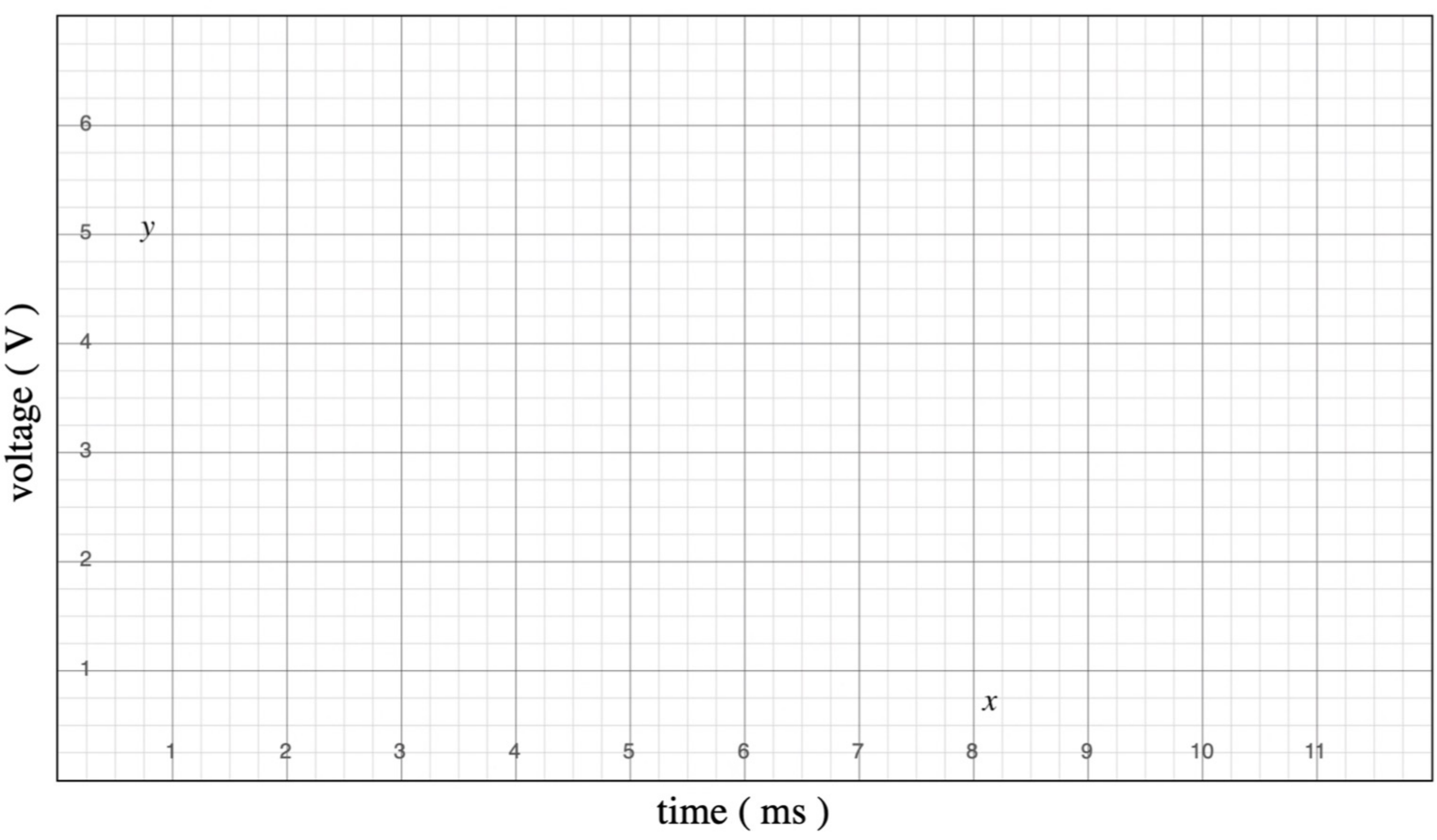
(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.

(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.

a

b



4. Linear Algebra Review

For the following matrices, find the following properties:

i. What is the column space of the matrix?

ii. What is the null space of the matrix?

iii. What are the eigenvalues and corresponding eigenspaces for the matrix?

(a) $\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$

For this matrix you are told that the eigenvalues are: 2, 1, and 0.